Different Approaches to Risk Estimation in Portfolio Theory

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Almost forty years ago, Sharpe [1966] developed a measure then called reward-to-variability for evaluating and predicting the performance of mutual fund managers. Under the name of the Sharpe ratio, it has become one of the most popular indexes in practical application.

Although the Sharpe ratio is fully compatible with normally distributed returns (or, in general, with elliptical returns), it will lead to incorrect investment decisions when returns present kurtosis or skewness (see, among others, Leland [1999] and Bernardo and Ledoit [2000]). Specifically, returns on assets exhibit heavy tails, and many researchers recognize the limitations of this performance measure (see Ortobelli et al. [2003]).

Several alternatives to the Sharpe ratio for optimal portfolio selection have been proposed such as the minimax ratio, the stable ratio, the mean absolute deviation ratio, the Farinelli-Tibiletti ratio, and the Sortino-Satchell ratio; see Young [1998], Ortobelli et al. [2003a, 2003b], Farinelli and Tibiletti [2003a, 2003b], Sharpe [1994], Dowd [2001], Sortino [2000], Pedersen and Satchell [2002], Szegö [2004], and Uryasev [2000]. All these performance measures are theoretically valid and lead to different optimal solutions, so it is unclear how an investor should decide on a criterion.

Our goal is to find a criterion whose application would lead to correct investment decisions in the case of heavy-tail distributed returns. We use an ex post analysis considering recent historical data. In our study period, stock returns were volatile and far from normally (Gaussian) distributed. We suggest new mean risk models that perform better than other models in the case of non-normal return distributions. Thus, the new performance measures take into account a more realistic behavior of stock return distributions. Using nine assets in the German market, we show that the Sharpe ratio does not perform as well as other criteria or the new ones introduced here.

RISK MEASURES

Before Markowitz [1959], there had been no attempt to quantify risk. Financial risk was considered a correcting factor for expected return. The Markowitz mean-variance framework was an excellent beginning, with the main innovation being measurement of the dependence structure of returns while accounting for their correlation. The dependence structure represented by the variance-covariance matrix, which quantifies the impact on portfolio selection of each asset with respect to the others, exemplifies the importance of diversification.

Sharpe's [1966] capital asset pricing model (CAPM) extended the Markowitz theory to an equilibrium theory. The simplicity and the intuitive appeal of mean-variance analysis and the equilibrium results attracted considerable research attention on generality and extendibility. Markowitz theory also found its way into the basis of arbitrage theory and stochastic dominance analysis.

Mean-variance theory has survived both theoretical criticism and empirical rejection. Roll [1977, 1978, 1979] was the first to understand the weaknesses of the theory and its empirical deficiencies. Alternative mean risk models have been proposed since then in response.

Many researchers have investigated the best risk measure to be applied in portfolio selection. See, for example, the mean-lower partial moment approach of Markowitz [1959], Bawa [1977], and Fishburn [1977]; the mean-Gini portfolio analysis of Yitzhaki [1982] and Shalit and Yitzhaki [1984]; the mean-absolute deviation (MAD) approach of Konno and Jamazaki [1991]; the mean-stable dispersion analysis of Ortobelli et al. [2003, 2004a]; and the mean-VaR models of Uryasev [2000] and Giacometti and Ortobelli [2004]. These mean risk approaches admit a CAPM approach, and can be easily extended to a multiparameter framework according to stochastic dominance rules because they are a subcase of the analysis proposed by Ortobelli [2001] and Ortobelli et al. [2003].

In a mean risk world, we can describe the uncertainty of future wealth with two summary statistics: a risk measure, and the mean representing reward. Various portfolio selection approaches differ in the risk measure used. In particular, we can distinguish two distinct categories of risk measures used in portfolio theory: dispersion measures, and safety risk measures. These categories differ in their consistency with stochastic dominance order.

Absolute Deviation

The absolute deviation of a random variable is the expected absolute value of the difference between the random variable and its mean. The absolute deviation is a linear risk measure consistent with a stochastic dominance order.*

The primary aim of the mean absolute deviation (MAD) portfolio optimization model of Konno [1990] and Konno and Yamazaki [1991] is overcoming the limitations of the CAPM. They propose a linear optimization problem based on the absolute deviation that improves and accelerates the computation of optimal portfolios. Because of its computational simplicity in solving portfolio selection problems, the absolute deviation has been popular in the last decade.

Stable Dispersion Measure

The stable dispersion measure is the scale parameter of a stable Paretian distribution (also called the α-stable distribution). Therefore, the mean-stable dispersion approach is a distributional approach based on the assumption that returns are stable Paretian distributed.

The adoption of stable modeling is undoubtedly one of the most interesting and promising ideas to arise in finance. While alternative models based on other non-Gaussian distributions appear in the literature, the stable assumption has unique distinctive characteristics that make it an ideal candidate. The functional central limit theorem for normalized sums of independent and identically distributed random variables theoretically justifies the stable Paretian approach.

Excess kurtosis found by Mandelbrot [1963] and Fama [1965] led them to reject the normality assumption and propose the stable Paretian distribution as a statistical model for asset returns. Ziemba [1974] and Rachev and Mittnik [2000], among others, extended these early studies of asset returns, consolidating the stable Paretian hypothesis.

The stable dispersion measure is consistent with a stochastic order, and it can be obtained as a function of a centered moment. Thus, in some sense, this risk measure can be seen as a generalization of absolute deviation.

Gini Risk Measure

The Gini risk measure, or Gini's mean difference, is a statistic used widely to measure income inequality. It is an index of the variability of a random variate, based on the expected value of the absolute difference between every pair of realizations. It admits a geometric interpretation, and has been used for 80 years by economists to describe economic and social conditions. Yitzhaki [1982] and Shalit and Yitzhaki [1984] have proposed a mean-Gini portfolio selection theory that is consistent with a stochastic dominance rule and presents many analogies with mean-variance analysis.

An advantage is that the portfolio optimization problem with the Gini risk measure is linear. Linearity simplifies the complexity of the portfolio choice problem; this is a major advantage over traditional risk measures. By using the Gini risk measure, we can associate with each portfolio a concentration curve that classifies its risk aversion. Therefore, it is possible to identify defensive and aggressive portfolio selection strategies depending on the location of their concentration curves.

Lower Partial Moment

All the dispersion measures we have described are measures of uncertainty; uncertainty, however, is not necessarily risk. Unlike safety risk measures, dispersion measures consider both positive and negative deviations from the mean as a potential risk. That is, overperformance relative to the mean is penalized just as much as underperformance.

To address this anomaly, Markowitz [1959] proposed the semivariance as a risk measure. A natural extension of semivariance is the lower partial moment risk measure, also called *downside risk* or *the probability-weighted function of deviations below a specified target return* (see Bawa [1977] and Fishburn [1977]).

This risk measure depends on two parameters: 1) a power index that is a proxy for the investor's degree of risk aversion, and 2) the target rate of return that is the return that must be earned at a minimum to accomplish the funding of the plan within the cost constraint. Moreover, considering the expected moment of deviations above a specified target return, called the *upper partial moment*, we obtain a measure of reward that can be used to value the excess of return.

Value at Risk

One of the most important tasks of a financial institution is the evaluation of exposure to the market risks that arise from changes in prices of equities, commodities, exchange rates, and interest rates. Dependence on market risks can be measured by changes in the portfolio value, or profits and losses.

A common methodology for estimation of market risks is value at risk (VaR_ϑ). Its primary characteristic is that it synthesizes in a single value the possible losses that could occur with a given probability $(1 - \vartheta)$ in a given temporal horizon. This feature, together with the (very intuitive) concept of maximum probable loss, allows the non-expert investor to figure out how risky a position is and the appropriate corrective strategies.

VaR models are increasingly adopted. Financial institutions with significant trading and investment volumes use a VaR methodology in their risk management operations.

Conditional Value at Risk

To select a minimal set of properties that a risk measure must satisfy to be a coherent risk measure, Artzner et al. [1999] define with remarkable financial insight the so-called axioms of coherency, and find the explicit representation of any coherent risk measure. In practice, a coherent risk measure recognizes that: 1) the wealth at risk declines when we add an amount of riskless wealth; 2) more wealth is preferred to less wealth; 3) the aggregated risk of two investments is lower than the sum of the two associated single risks; and 4) when the wealth at risk is multiplied by a positive factor, risk must grow also with the same proportionality. These properties are mathematically formalized in axioms that ensure the coherency of a risk measure.

Even if there is no doubt that VaR_ϑ provides useful information, it is not a coherent risk measure, and it cannot offer an exhaustive representation of an investor's preferences. To overcome these limitations and problems, Artzner et al. [1999] propose the *conditional value at risk* (CVaR) as an alternative risk measure. The CVaR, also called *expected shortfall* or *expected tail loss*, measures the expected value of portfolio returns, given that the VaR has been exceeded.

CVaR is a coherent risk measure, and portfolio selection with the expected shortfall can be reduced to a linear optimization problem. Moreover, if we apply the CVaR concept symmetrically to the opposite part of the portfolio returns distribution, we get a measure of portfolio reward.

Mini-Max

The mini-max risk measure was first used in portfolio selection problems by Young [1998]. It represents the maximum loss over all past observations. It is possible to show that the mini-max risk measure is an extreme and special case of the CVaR. Therefore, the mini-max risk measure satisfies all the properties of the expected tail loss.

Power Conditional Value at Risk

To take into account the investor's degree of risk aversion, we have introduced *power CVaR*. This new risk measure is the expected value of the excess return raised to a certain power, given that some threshold (usually the VaR) has been exceeded by the excess return. Therefore, this risk measure depends on an additional parameter, *the power index*, which varies with the investor's degree of risk aversion.

PERFORMANCE MEASURES

For each risk measure, there is a performance measure to identify superior, ordinary, and inferior performance. In particular, the ratio between the expected excess return and the relative risk measure is a performance measure $\rho(.)$ that investors wish to maximize. The maximization of the performance measure $\rho(.)$ determines the so-called market portfolio, because it represents the benchmark for the market. Therefore, the market portfolio found for each performance measure $\rho(.)$ is based on a diverse risk perception and sometimes on a different reward perception.

Even if most researchers have tried to find the "right" risk measure, our empirical analysis shows that the right reward perception is also crucial to determine a desirable risky investment strategy. We do not believe there is a perfect performance measure, because a performance measure depends on only two parameters that summarize the complexity of all admissible choices.

Some performance measures, however, take into account typical investor opinions better than others. As a matter of fact, most decision-makers have some common beliefs. For example, they prefer more wealth than less, and they are generally risk-averse.

For this reason, most of the risk measures proposed in the literature are consistent with at least one stochastic dominance order. Thus, if we minimize a given risk measure (consistent with a stochastic order) for a fixed expected return level, we find a portfolio that is optimal for some investors. This does not mean that this portfolio is the best choice for all decision-makers who want a return with the same mean.

Similarly, when we maximize a performance measure, we determine the market portfolio that best represents a predisposition to risk and an inclination to invest for a given category of investors. These investors do not necessarily represent all the agents in the market, but we see a common interest for all decision-makers, which is to maximize final wealth.

Therefore, in our empirical analysis, we emphasize the performance measures that maximized final wealth in our sample period: the Sharpe [1966, 1994] ratio; the stable ratio (see Ortobelli et al. [2003]); the MAD ratio (see Konno and Yamazaki [1991]); the Gini ratio (see Yitzhaki [1982] and Shalit and Yitzhaki [1984]); the Sortino-Satchell ratio (see Sortino [2000], Sortino and Satchell, [2001], and Pedersen and Satchell [2002]); the $VaR_{99\%}$ ratio (see Favre and Galeano [2002] and Martin, Rachev, and Siboulet [2003]); the $CVaR_{99\%}$ ratio (see Martin, Rachev, and Siboulet [2003]); the mini-max ratio (see Young [1998]); and the Farinelli-Tibiletti ratio [2003a, 2003b]. We also introduce two new performance ratios: the Rachev ratio and the Rachev generalized ratio.

Rachev Ratio

The Rachev ratio (R-ratio) is the ratio between the CVaR of the opposite of the excess return at a given confidence level and the CVaR of the excess return at another confidence level. That is:

$$\rho(x'r) = \frac{CVaR_{(1-\alpha)\%}(r_f - x'r)}{CVaR_{(1-\beta)\%}(x'r - r_f)} \tag{1}$$

or, alternatively:

$$\rho(x'r) = \frac{ETL_{\alpha\%}(r_f - x'r)}{ETL_{\beta\%}(x'r - r_f)} \tag{2}$$

where α and β belong to (0, 1). Here, if X is a return of a given portfolio, then $L = -X$ represents the relative loss, and $ETL_{\alpha\%}(X) = E(L/L > VaR_{\alpha\%})$ is the expected tail loss where $VaR_{\alpha\%}$ is defined by $P(L > VaR_{\alpha\%}) = \alpha$, and α belongs to (0, 1). In our empirical analysis, we consider this ratio for different choices of the parameters α

and β:

Rachev ratio1 (when $\alpha = \beta = 0.01$) (R1-ratio)
Rachev ratio 2 (when $\alpha = \beta = 0.05$) (R2-ratio)
Rachev ratio 3 (when $\alpha = 0.5$, $\beta = 0.01$) (R3-ratio)

The Rachev ratio is a very flexible performance measure that generalizes the $CVaR_{99\%}$ ratio (the STARR ratio). In many cases, a flexible measure of reward, together with a coherent definition of risk, lets us value the investor's performance correctly. These characteristics represent the principal theoretical advantages of the R-ratio with respect to the other performance ratios proposed in the literature.

Rachev Generalized Ratio

The Rachev generalized ratio (RG-ratio) is the ratio between the power CVaR of the opposite of the excess return at a given confidence level and the power CVaR of the excess return at another confidence level. That is:

$$\rho(x'r) = \frac{ETL_{(\gamma,\alpha\%)}(r_f - x'r)}{ETL_{(\delta,\beta\%)}(x'r - r_f)} \tag{3}$$

where $ETL_{(\gamma,\alpha\%)}(X) = E(\max(L, 0)^{\gamma}/L > VaR_{\alpha\%})$ is the power CVaR of X, and γ is a positive constant. In the empirical study we analyze the RG-ratio with parameters $\gamma = \delta = \alpha/2$, and $\alpha = \beta = 0.5$. The main advantage of the RG-ratio relative to the R-ratio is given by the power indexes γ and δ that characterize the investors' aversion to risk.

Further analysis, discussion, and detailed studies of the characteristics of the performance ratios can be found in Ortobelli, Rachev, Biglova, Stoyanov, and Fabozzi [2004].

EMPIRICAL COMPARISON

For ex post comparison of several performance ratios, we assume that every day all wealth is invested in the portfolio that maximizes a given performance measure $\rho(.)$. There are two cases:

1. There is no riskless asset (mathematically, this can be expressed assuming that the risk-free rate of return is equal to zero, i.e. $r_f = 0$).
2. The rate of return of the riskless asset r_f is represented by LIBOR (daily observations).

Thus, in the most general case, we analyze the problem of optimal allocation among $n + 1$ assets; n of those assets are risky with returns (continuously compounded) $r = [r_1, r_2, \dots, r_n]'$, and the $(n + 1)$-th asset is the risk-free asset (or a market index) with return r_f. No short-selling is allowed, i.e., the portfolio weights x_i corresponding to the risky assets for every $i = 1, \dots n$ and the weight $\lambda = 1 - \sum_{i=1}^{n} x_i$ of the risk-free asset are greater than or equal to zero. We assume all portfolios are uniquely determined by the mean and a risk measure consistent with some stochastic dominance order. Under these assumptions, most of the risk measures proposed in the literature are equivalent (see Ortobelli, Rachev, Biglova, Stoyanov, and Fabozzi [2004]).

In addition, investors will choose an optimal portfolio that is a linear combination of the risk-free asset and an optimal risky portfolio. The optimal risky portfolio is given by the portfolio that maximizes the performance measure $\rho(.)$. Generally, this measure is a ratio between the expected excess return and a risk measure of portfolio return. Thus, for any performance measure $\rho(.)$, we compute the market portfolio $x_M'r$ that is the solution to the optimization problem:

$$\max_x \rho(x'r)$$
$$s.t.$$
$$\sum_{i=1}^{n} x_i = 1; x_i \geq 0; i = 1, \dots, n \tag{4}$$

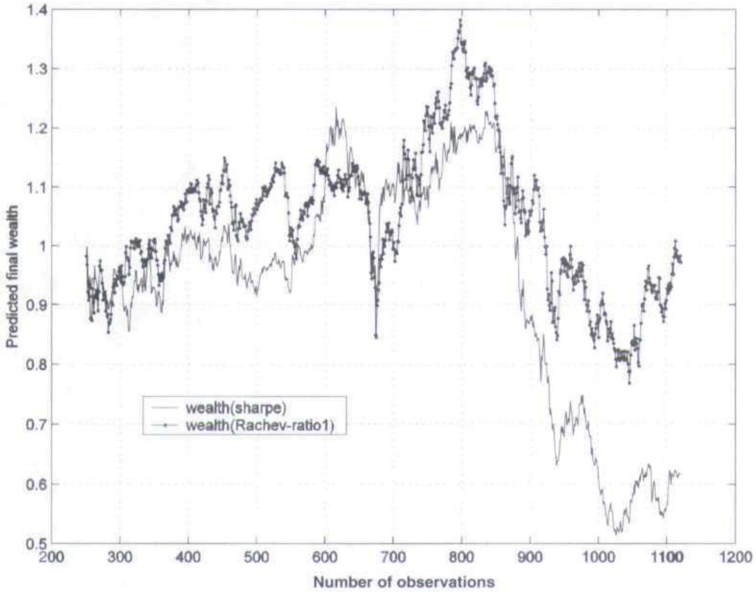
For different performance measures, we generally obtain different optimal risky portfolios because all the admissible choices are not uniquely identified by only two parameters. Therefore, the market portfolio composition $x'_M = [x_{1,M}, \dots, x_{n,M}]$ found for each performance measure $\rho(.)$ is based on a different risk perception and sometimes on a different reward perception. We could propose a portfolio selection theory that depends on more than two parameters (see Ortobelli [2001]), but the choice problem becomes more complex.

Once we have computed the market portfolio as a solution of the optimization problem in (4), we compare the sample paths of the final wealth obtained using the different approaches.

We use daily data for the period January 27, 1999, through June 30, 2003. We examine the optimal allocations among the risk-free asset (that is, LIBOR when we assume the presence of a risk-free asset) and nine risky assets in the German market (Adidas Salomon AG, Basf AG, Bayerishe Motoren Werke AG, Continental AG,

EXHIBIT 1

Predicted Final Wealth of Market Portfolio—Risk-Free Rate = 0



Ratio	*Value of portfolio at day 1120*
Sharpe Ratio	*0.616240415853423*
Mini-Max Ratio	*0.804530138619293*
Stable Ratio	*0.578139498466928*
MAD Ratio	*0.628476996045711*
Gini Ratio	*0.625304181881120*
Sortino-Satchell Ratio	*0.617976033633241*
Farinelli-Tibiletti Ratio	*0.839552761793775*
$VAR_{99\%}$ Ratio	*0.605072926895815*
$CVaR_{99\%}$ Ratio (STARR Ratio)	*0.776588630065149*
Rachev Ratio1	*0.972486708253634*
Rachev Generalized Ratio	*0.753680291712404*

Bayer AG, Hoechst AG, Fresenius Medical Care AG, MAN AG, and Henkel KGAA).

Generally in the literature, there are different definitions of return. Let us denote by $r_{i,t}$ the return of the i-th asset at time t. Observe that the results in the optimization problem could change, depending on whether returns are defined as a percentage change:

$$r_{i,t} = \frac{S_{i,t} - S_{i,t-1}}{S_{i,t-1}}$$

(where $S_{i,t}$ is the price of the i-th asset at time t), or continuously compounded returns:

$$r_{i,t} = \log\left(\frac{S_{i,t}}{S_{i,t-1}}\right)$$

Thus, we analyze the two cases separately.

Percentage Change Return

Let us suppose that decision-makers invest their wealth and purchase the market portfolio. Considering that each investor has a different risk perception and a different excess return perception, the market portfolio is determined by maximizing one of the performance measures. To compare the efficiency of alternative performance measures, we consider the sample path of the final wealth obtained from the various approaches.

We assume the investor has an initial wealth of $W_0 = 1$ on January 27, 2000. Thus, every day and for each performance measure $\rho(.)$, we solve the optimization problem in (4) using the observations from the prior year (the prior 250 observations) for calibration purposes. Once we determine the market portfolio $x'_{M(n)}$, the final wealth after n days is given by

$$W_n = W_{n-1}(1 + x'_{M(n)} r_{(n)}) \tag{5}$$

where $r_{(n)}$ is the vector of returns gained between the $n-1$ and the n-th day starting from January 27, 2000. We assume either r_f = LIBOR or $r_f = 0$.

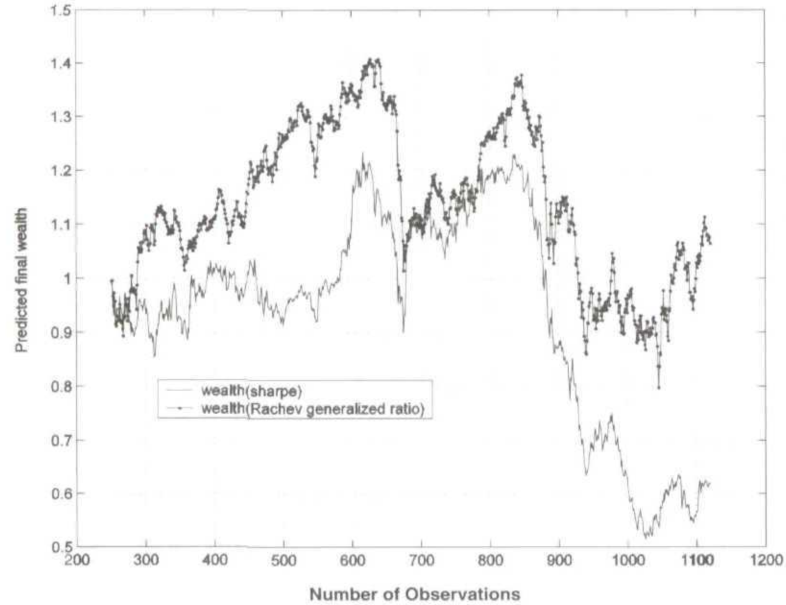
Exhibit 1 presents graphs of the predicted final wealth of the market portfolio for the best ratio and for the Sharpe ratio as well as values of the final wealth W_{870} of the portfolio when $r_f = 0$. The abbreviation W_{870} denotes that we consider the wealth accumulated after 870 periods of optimization.

Exhibit 2 presents the same information when we assume r_f = LIBOR.

These exhibits show that the R1-ratio and the RG-ratio sample paths dominate the Sharpe ratio sample path. Therefore, we can conclude that for our sample of stocks and our study period investing in the market portfolio every day, assuming either the R1-ratio approach or the RG-ratio approach, we become wealthier in the end. It follows that the R1-ratio and RG-ratio models perform better than the classic Sharpe ratio.

EXHIBIT 2

Predicted Final Wealth of Market Portfolio—Risk-Free Rate = LIBOR



Ratio	*Value of portfolio at day 1120*
Sharpe Ratio	*0.617451929444473*
Minimax Ratio	*0.775256808505809*
Stable Ratio	*0.536188972808025*
MAD Ratio	*0.638236615004889*
Gini Ratio	*0.634635872560578*
Sortino-Satchell Ratio	*0.617522512797628*
Farinelli-Tibiletti Ratio	*0.989935546412939*
$VAR_{99\%}$ Ratio	*0.630538999681293*
$CVaR_{99\%}$ Ratio (STARR Ratio)	*0.723743228580052*
Rachev Ratio1	*1.006171785779270*
Rachev Generalized Ratio	*1.063973580505089*

Most of the other approaches are significantly different from the mean-variance one, even though we see the optimal portfolio weights are not always significantly different. This result implicitly supports the hypothesis that the new ratios, the R1-ratio and the RG-ratio, appear to capture the distributional behavior of the data (typically the component of risk due to heavy tails) better than the classic mean-variance model.

Continuously Compounded Return

Assuming continuous compounding, we examine the same optimal allocation among the risk-free return r_f and nine German assets during the study period (1,120 observations). We examine portfolio selection with and without the risk-free asset.

We assume the investor has an initial wealth equal to 1 and initial cumulative return $CR_0 = 0$ on January 27, 2000. Thus, every day and for each performance measure $\rho(.)$, we have to solve the optimization problem in (4) using the observations of the prior year (the prior 250 observations) for calibration purposes. Once we determine the market portfolio $x'_{M(n)}$, the cumulative return CR_n after n days is given by

$$CR_n = CR_{n-1} + x'_{M(n)} r_{(n)} \tag{6}$$

where $r_{(n)}$ is the vector of continuously compounded returns gained between the $n-1$ and the n-th day starting January 27, 2000.

Exhibit 3 presents sample paths of cumulative returns for the best ratio and for the Sharpe ratio and values of cumulative returns for some other ratios (after 870 periods of optimization and return accumulation) when there is no risk-free asset. The plots show the R1-ratio always performs better than the Sharpe ratio, and it yields the maximum total realized return (equal to 9.583%) at the end of the period examined.

To see the relevance of this result, we compare the cumulative returns of the R1-ratio with one of the most representative German indexes, the DAX 30. Then we consider the sample path of cumulative returns for the DAX 30 during the period examined. That is

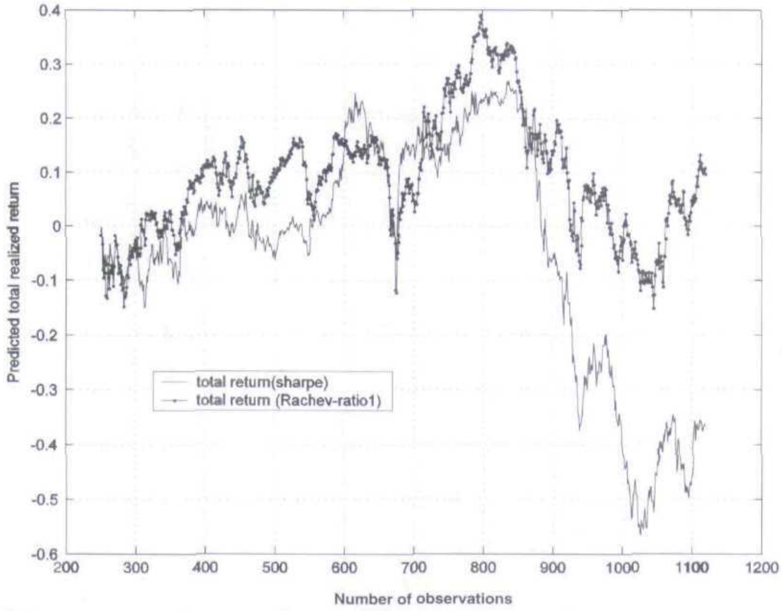
$$CR_n(DAX30) = \log\left(\frac{S^{dax}(n)}{S^{dax}(0)}\right)$$

where $S^{dax}(0)$ is the value of the DAX 30 on January 27, 2000, and $S^{dax}(n)$ is its value n days from that date.

Exhibit 4 shows that the forecast based on the R1-ratio produces the best result for the entire period, realizing a higher return than the DAX 30. If investors had put all their wealth in the DAX 30 on January 27, 2000, at the end of the period on June 30, 2003, they would have lost 16.606% of the initial wealth. If they had instead picked a portfolio using the R1-ratio every day, they would have realized a return of 9.583%.

EXHIBIT 3

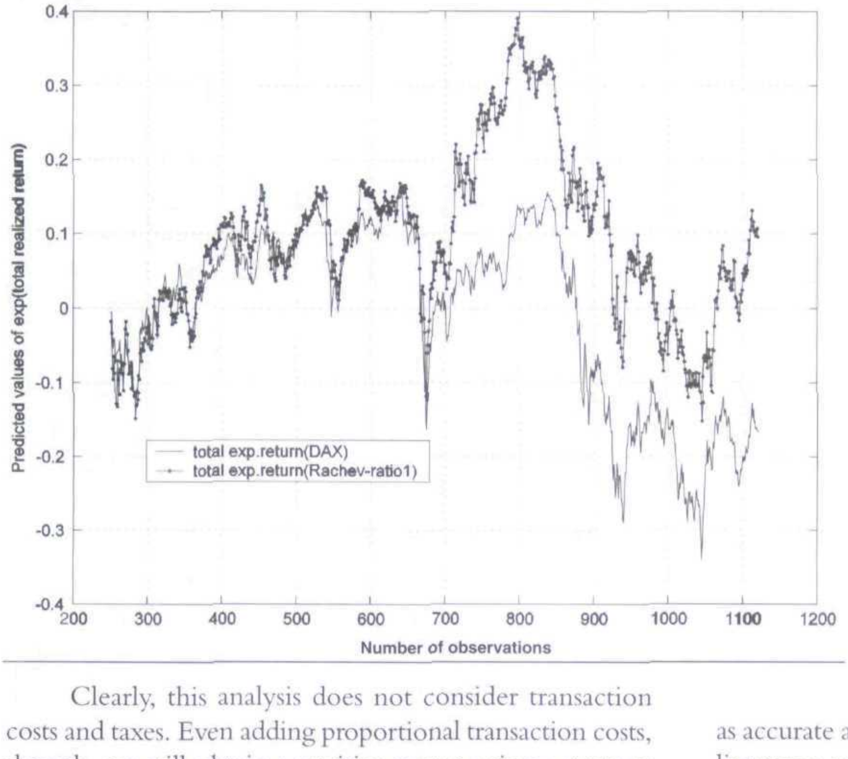
Change in Predicted Total Realized Return—Risk-Free Rate = 0



Ratio	*Value of portfolio at day 1120*
Sharpe Ratio	*-0.366585953821371*
Minimax Ratio	*-0.098776164821371*
Stable Ratio	*-0.437531222524888*
MAD Ratio	*-0.344087473821371*
Gini Ratio	*-0.350627123821371*
Sortino-Satchell Ratio	*-0.359212613821371*
Farinelli-Tibiletti Ratio	*0.010434873821371*
$VAR_{99\%}$ Ratio	*-0.395411563821371*
$CVaR_{99\%}$ Ratio (STARR Ratio)	*-0.142927703821371*
Rachev Ratio1	*0.095833163360289*
Rachev Generalized Ratio	*-0.191643126275180*

EXHIBIT 4

Change in Predicted Total Realized Return—Comparison with DAX 30



Clearly, this analysis does not consider transaction costs and taxes. Even adding proportional transaction costs, though, we still obtain a positive return using a strategy based on the R1-ratio.

As one would expect, the final wealth increases when we add a riskless asset. Exhibit 5 presents sample paths of cumulative returns for the best ratio and for the Sharpe ratio and values of predicted total realized returns for other ratios when we assume r_f = LIBOR. The RG-ratio produces the best performance.

Exhibit 6 shows that the R1-ratio sometimes yields a better performance than the RG-ratio. In any case, both the R1-ratio and the RG-ratio realize a cumulative return greater than that of the German index. As a matter of fact, at the end of the period examined, the cumulative return obtained using the R1-ratio is 12.933%, while the return received using the RG-ratio is 17.375%.

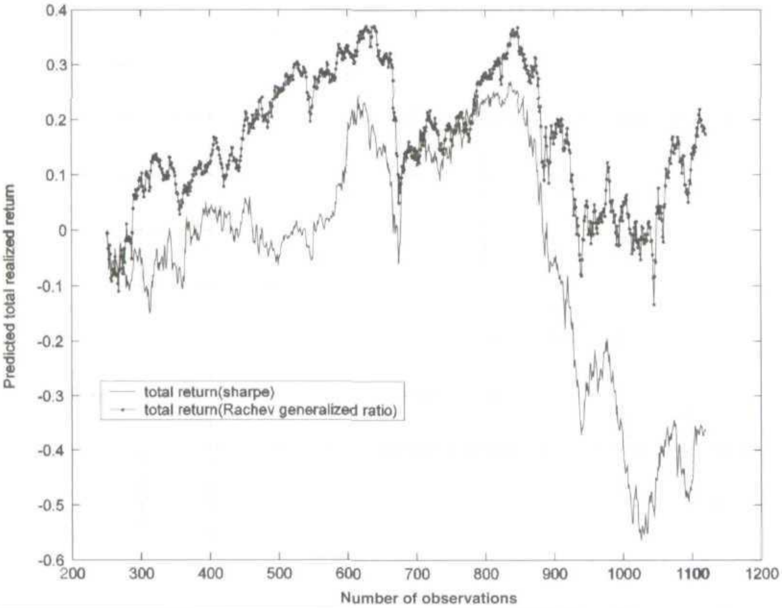
CONCLUDING REMARKS

This work compares alternative portfolio selection models based on different risk measures, including the most popular one, the Sharpe ratio.

Our empirical analysis of all these ratios suggests that the Sharpe ratio forecasts are not as accurate as other performance measures proposed in the literature such as the mini-max ratio or the Farinelli-Tibiletti ratio. We suggest two new performance measures (the Rachev ratio and the Rachev generalized ratio) as good criteria for portfolio optimization because they

EXHIBIT 5

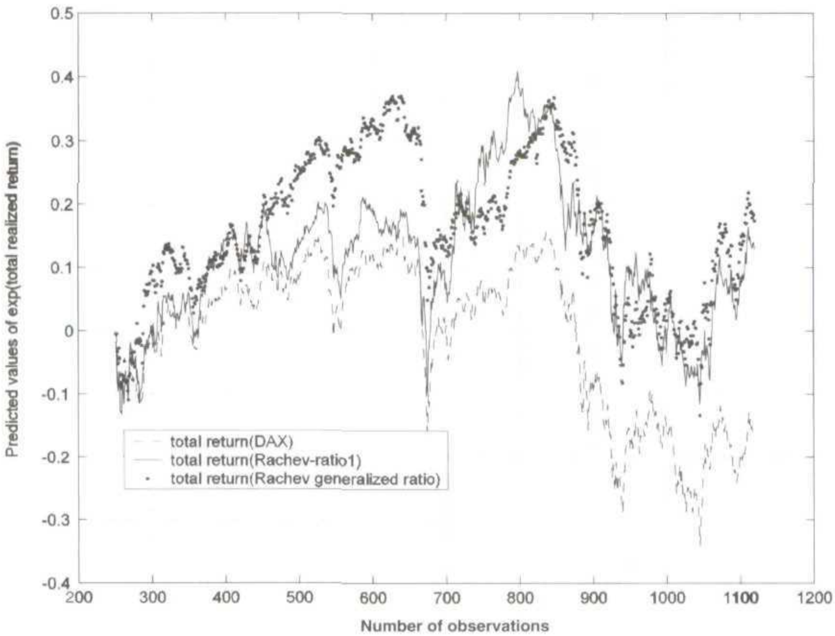
Change in Predicted Total Realized Return—Risk-Free Rate = LIBOR



Ratio	Value of portfolio at day 1120
Sharpe Ratio	-0.364645423821371
Minimax Ratio	-0.135864083821371
Stable Ratio	-0.520872263821371
MAD Ratio	-0.328669043821371
Gini Ratio	-0.335808033821371
Sortino-Satchell Ratio	-0.360183193821371
Farinelli-Tibiletti Ratio	0.111626853821371
$VAR_{99\%}$ Ratio	-0.344402793821371
$CVaR_{99\%}$ Ratio (STARR Ratio)	-0.199963063821371
Rachev Ratio1	0.129330688520624
Rachev Generalized Ratio	0.173757447754619

EXHIBIT 6

Changes in Predicted Total Realized Return—DAX and R1- and R6-Ratios



yield the best performance for investors in the period examined.

ENDNOTES

Svetlozar Rachev gratefully acknowledges research support by grants from the Division of Mathematical, Life and Physical Sciences of the College of Letters and Science at the University of California, Santa Barbara; the Deutschen Forschungsgemeinschaft; and the Deutscher Akademischer Austausch Dienst. Sergio Ortobelli has been partially supported by grants from COFIN (2003, 2004) and CNR-MIUR-Legge 95/95.

*Generally, we say that a risk measure is consistent with a stochastic order if, for a given category of investors (either risk-averse or who prefer more than less) the preference of random wealth W_1 to random wealth W_2 implies that wealth W_1 is less risky than wealth W_2.

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